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On Galvanomagnetic Size Effects in Metals

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The importance of open electron trajectories formed by specular reflections of charge carriers by the sample boundary to the metal electric conductivities in the strong magnetic field H is analyzed. It is shown that the electric conductivity of the near-surface layer σ_{skin} is rather sensitive to the state of the conductor surface. The electron–hole Umklapp processes during the surface scattering of charge carriers do not change the dependence $\sigma_{\text{skin}}(H)$, while skipping from the closed Fermi surface section to the open one is able to affect σ_{skin} essentially only in bulk samples. The method is proposed to restore the indicatrix of conduction electron scattering by the sample boundary through an experimental investigation of the Sondheimer effect and the static skin effect.

KEY WORDS: Static skin effect; Umklapp surface scattering; indicatrix of conduction electron scattering; open electron trajectories; high magnetic field.

The theory of galvanomagnetic phenomena in metals developed by I. M. Lifshitz and his co-workers[1,2] for an arbitrary energy–momentum relationship for charge carriers lays the groundwork for investigations of the topology of the electron energy spectrum. The sensitivity of galvanomagnetic characteristics of metals to the Fermi surface (FS) topology is accounted for by the fundamental difference between the dynamics of free electrons and that of conduction electrons belonging to open FS sections whose drift does not coincide with the direction of the magnetic field **H**. The presence of a thin layer of such sections essentially changes the electric conductivity of a metallic sample in high magnetic fields when the trajectory curvature radius r of an electron is much smaller than its mean free path l. As the thickness of the open orbit layer related to the Fermi momentum exceeds the $(r/l)^2$ value,

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the dependence of the bulk conductor resistance on a high magnetic field changes. Therefore, the appearance or disappearance of the layer of FS open sections due to variation of the magnetic field orientation with respect to crystallographie axes causes a sharp anisotropy of the transverse magnetoresistance of single crystals (the current density is $\mathbf{j} \perp \mathbf{H}$), which either saturates at $r/l \to 0$ or increases quadratically with the magnetic field. In polycrystalline samples the presence of open orbits leads to a linear dependence of the resistance on H,[2,3] or to a close to linear dependence.[4,5]

In thin conductors whose thickness d is smaller than l, in addition to the sharp anisotropy of the transverse magnetorisistance caused by the FS topology, there are a number of size effects whose investigation gives more detailed information on charge carrier properties in metals. The Sondheimer effect specific to thin conductors[6]—the oscillatory dependence of resistance on the sample thickness and high magnetic field ($r \ll d$)—appears to be informative in studies of conduction electron spectrum similarly as the resonance and quantum oscillation effects are.[7–9] The static skin effect[10–12] offers possibilities of a detailed investigation of one of the mechanisms of electron current dissipation connected with the interaction between charge carries and the metallic surface. The concentration of dc electric current near the sample surface is accounted for by a higher mobility of conduction electrons colliding with the conductor surface as compared to that of the charge carriers experiencing no sample boundaries. In smooth-surface conductors even with the isotropic energy–momentum relationship for conduction electrons an open trajectory of the electron motion can be formed at the expense of its specular reflections by the sample boundary. If the conductor surface has a flat face and the magnetic field is parallel to this face, the electrons skipping along these trajectories over the sample surface give a determining contribution to the electric conductivity of the metal whose resistance increases infinitely with the magnetic field. The factors causing the resistance growth—viz., the compensation of electron and hole volumes, the presence of open FS sections, or the magnetic breakdown—are of no essential importance in this case. The core of the sample does not participate practically in its electric conductivity, and the electric current is almost completely concentrated in the near layer with thickness of the order of r.

The conduction electron drift along the artificially formed open trajectories is restricted by the mean free path l, and the near surface layer conductivity σ_{skin} coincides by the order of magnitude with the bulk metal conductivity σ_0 at $H = 0$. But charge carriers can be specularly reflected only with a thermodynamically equilibrium nondefective surface of the crystal.[13,14] Only then does the sample resistance inversely proportional to the current layer thickness increase linearly with the magnetic field. The real surface of a conductor is not quite perfect and the roughness available would cause the effective mean free path of charge carriers related to σ_{skin} as

$$\sigma_{\text{skin}} = l_{\text{eff}} \sigma_0 l^{-1} \tag{1}$$

to tend at $l \to \infty$ to a finite value determined by the distance covered by an electron during the time interval between its collisions with the sample boundary and the surface roughness. As the magnetic field increases, both the skin-layer thickness and the effective mean free path of the conduction electrons interacting with the sample boundary decrease, since $\Delta x_s \simeq r$ and the electron has to collide with the conductor surface more frequently. As a result, under conditions of the static skin effect, the resistance of rough-surface conductors is proportional to H^2, and the proportionality coefficient is dependent on the scattering properties of the metal surface.

1. It is not difficult to estimate l_{eff} assuming that charge carriers collide only with thermodynamically equilibrium areas of the sample surface and do not get into intermediate areas, where correlation between the incident $\mathbf{p}_-$ and reflected $\mathbf{p}_+$ electron momenta most likely is lost. This is achieved by averaging the expression for electric current density

$$\mathbf{j}(\mathbf{r}) = \int \frac{2d^3\mathbf{p}}{(2\pi\hbar)^3} e\mathbf{v} f_0(\varepsilon - \Delta\varepsilon) \tag{2}$$

over the conductor volume.

Here e, $\mathbf{v}$, $\mathbf{p}$, ε, f_0 are the charge, velocity, momentum, energy, equilibrium Fermi function of conduction electron distribution, $\hbar$ is the Planck constant, $\Delta\varepsilon$ the energy gained by a charge carrier in the electric field $\mathbf{E}(\mathbf{r})$

$$\Delta\varepsilon = \int_{\lambda_1}^{t} e\mathbf{v}(t')\,\mathbf{E}(\mathbf{r} + \mathbf{r}(t') - \mathbf{r}(t)) \exp\left(\frac{t'-t}{\tau}\right) dt'$$
$$+ \sum_{i=1}^{\infty} \int_{\lambda_{i+1}}^{\lambda_i} e\mathbf{v}(t')\,\mathbf{E}(\mathbf{r}(t') + \mathbf{r}_{i+1} - \mathbf{r}(\lambda_{i+1})) \exp\left(\frac{t'-t}{\tau}\right) dt' \tag{3}$$

Here $\tau = l/v$ is the mean free time of conduction electrons with respect to the intrabulk collisions, t (or t') is the time of a charge carrier motion in the electric and magnetic fields, λ_i are the moments of a charge reflection by the sample boundary at the points $\mathbf{r}_i$, which can be found using the equation

$$\begin{aligned} &\int_{\lambda_1}^{t} \mathbf{v}(t')\,dt' \equiv \mathbf{r}(t) - \mathbf{r}(\lambda_1) = \mathbf{r} - \mathbf{r}_1, \qquad \lambda_1 \leqslant t \\ &\mathbf{r}(\lambda_i) - \mathbf{r}(\lambda_{i+1}) = \mathbf{r}_i - \mathbf{r}_{i+1}, \qquad \lambda_{i+1} < \lambda_i, \qquad i \geqslant 1 \end{aligned} \tag{4}$$

It is clearly seen (Fig. 1) that the effective mean free path of the charge carriers interacting with a surface of great and randomly distributed roughness appears to be of the order of r. This effective mean free path occurs also in the model of pure diffuse reflection, where all the directions of the reflected electron motion are of equal probability. Thus, the conductor surface with randomly distributed roughness appears to be an additional

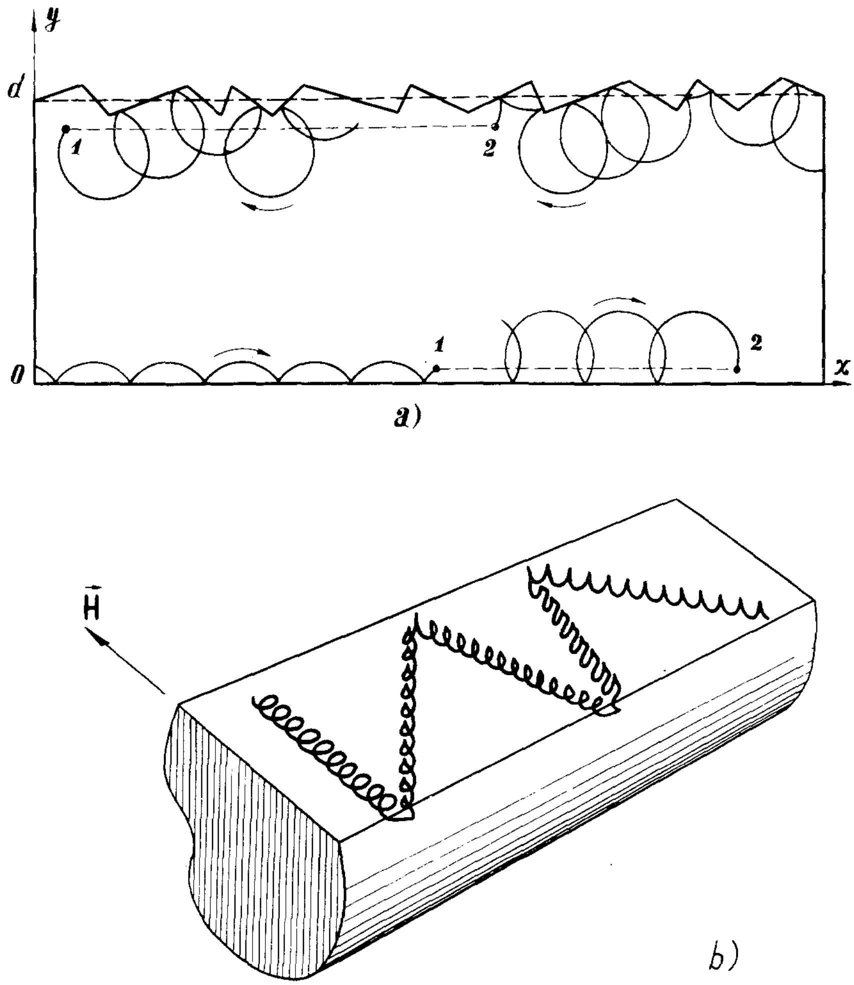

Fig. 1. Electron trajectories broken by specular reflections in a plate (a) and in wires with a flat face (b). Magnetic field is parallel to flat faces. The drifts of electrons in states 1 and 2 ($v_{x1} = -v_{x2}$) during the mean free time near a very rough upper surface of the plate are about the same and differ by an order of the mean free path near the lower smooth boundary of the sample.

scatterer of conduction electrons in the case of locally specular reflection of charge carriers, when their energy and momentum projection onto the plane contacting the metallic surface at the point r_i are conserved.

The effective mean free path of the charge carriers forming the skin layer in a plane-parallel plate can be found using the following interpolation formula:

$$l_{\text{eff}}^{-1} = l^{-1} + w/r \tag{5}$$

where w^{-1} stands for the number of collisions of an electron with the plate surface having randomly distributed roughness, after which the memory of the initial conditions is wiped off.

In wires whose surface has a flat face with wide d_s the conductivity of the skin layer is similar to that in the plate if $d_s \gg r/w$. In the opposite limiting case when the distance at which an electron "forgets" its initial conditions is much greater than d_s, the electric conductivity of the sample is sensitive to the state of a small area with width of the order of r of the wire surface contacting the flat face. If the roughness in this area is not small, then the main contribution to the electric conductivity of the skin layer is made by the conduction electrons slowly drifting along the direction of the magnetic field. In this case σ_{skin} is calculated in a mode similar to Fuchs' calculation[15] of the electric conductivity at $H = 0$ of a thin plate with rough faces and l_{eff} found from Eq. (5) acts as a mean free path

$$\sigma_{\text{skin}} = \sigma_0 \frac{d_s}{l} \ln \frac{l_{\text{eff}}}{d_s} \tag{6}$$

Before relating the experimental magnitude w to the state of the conductor surface, we shall analyze the importance of multichannel processes of specular reflection of charge carriers by a metallic surface to the skin-layer electric conductivity in the case of a perfect smooth boundary of the sample.

2. In contrast to free electrons, in metals there exist several channels of specular reflection of charge carriers by the sample boundary, i.e., with a given momentum $\mathbf{p}_-$ the condition

$$\varepsilon(\mathbf{p}_+) = \varepsilon(\mathbf{p}_-), \qquad [\mathbf{p}_+ \mathbf{n}] = [\mathbf{p}_- \mathbf{n}] \tag{7}$$

is fulfilled not with one, but with several values of the momentum $\mathbf{p}_+ = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N\}$, where N is the number of nonequivalent states of a reflected electron and $\mathbf{n}$ is the interior normal to the sample surface. The number of arrival channels with $\mathbf{vn} < 0$ coincides with that of specular reflection channels so that each of N vectors of $\mathbf{p}_+$ is in correspondance with N solutions $\mathbf{p}_- = \{\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \ldots, \tilde{\mathbf{p}}_N\}$ of Eq. (7).

Equation (2) for electric current density remains valid if Eq. (3) is averaged over all the possible channels of specular reflection of charge carriers by the sample boundary. In this form $f_0(\varepsilon - \Delta\varepsilon)$ coincides with the distribution function of conduction electrons $f(\mathbf{r}, \mathbf{p}) = f_0(\varepsilon) - \psi\, \partial f_0/\partial\varepsilon$ obtained through solving the kinetic Boltzmann equation

$$\frac{\partial\psi}{\partial t} + \mathbf{v}\,\frac{\partial\psi}{\partial\mathbf{r}} + \hat{W}\psi = e\mathbf{v}(t)\,\mathbf{E}(r) \tag{8}$$

in the τ approximation for the collision integral $\hat{W}\psi = \psi/\tau$ with the boundary condition

$$\psi_i(\mathbf{p}_+) = W_{ik}\psi_k(\mathbf{p}_-), \qquad W_{ik} = W_{ki} \tag{9}$$

where $\psi_i(\mathbf{p}_+) = \psi(\mathbf{p}_i)$, $\psi_i(-\mathbf{p}) = \psi(\tilde{\mathbf{p}}_i)$, and W_{ik} is the probability for an electron incident on the sample surface with the momentum $\tilde{\mathbf{p}}_k$ to choose the ith channel of specular reflection. The nonlinearity of the collision integral is mainly connected with heating by the conduction electron current and is negligible if the energy received by a charge carrier during the mean free time is much lower than that of the smearing of the Fermi distribution function. Under these conditions the nonlinear effects in the electric conductivity of thin conductors result only from the influence of the electric field and the intrinsic magnetic field of the current $\mathbf{H}'$ upon the ballistics of conduction electrons.[16] They may be analyzed through solving linearized Eq. (8)

$$\psi = \int_{\lambda_1}^{t} e\mathbf{v}(t')\,E(\mathbf{r} + \mathbf{r}(t') - \mathbf{r}(t))\exp\left(\frac{t'-t}{\tau}\right) dt'$$
$$+ F(\mathbf{r} - \mathbf{r}(t'))\exp\left(\frac{\lambda_1 - t}{\tau}\right) \tag{10}$$

These effects are essential only in the region of low magnetic fields. In the case $r \ll d$, the inhomogeneous magnetic field H' results in increasing the electric conductivity of the core of the sample due to the charge carrier drift along the vector $[\mathbf{H}, \nabla\mathbf{H}']$. The expression for σ_{skin} remains, however, practically unchangeable until $H'_{\max} \ll H$, i.e., $j \ll cH/d$. Therefore, we confine ourselves to a linear approximation in the electric field and treat the variables t and t' in Eqs. (3), (10), and (8) as the time of a conduction electron motion only in the constant and homogeneous field $\mathbf{H}$.

We shall assume that a conductor is a plane-parallel plate and the magnetic field is parallel to its faces. Then the static skin effect is the most pronounced.

a. Compensated Metal. In a high magnetic field an electron interacts only with one face of the plate, and with respect to single channel specular reflection the function $F(\mathbf{r}-\mathbf{r}(t))$ is invariable along the whole trajectory of the charge motion. In the process of multichannel reflection the conduction electron trajectory consists of different arcs corresponding to N specular reflection channels and the periodicity of the charge carrier motion is broken. In this case the boundary condition given by Eq. (9) is a set of N algebraic equations for F_i different functions

$$F_i = W_{ik}(A_k + F_k)\,\alpha_k^{-1}, \qquad 1 \leqslant i,\ k \leqslant N$$
$$A_k \equiv \int_{\lambda_1}^{\lambda_1+T_k} e\mathbf{v}(t')\,\mathbf{E}(\mathbf{r}_1 - \mathbf{r}(\lambda_1) + \mathbf{r}(t'))\exp\left(\frac{t'-\lambda_1}{\tau}\right) dt' \tag{11}$$

where $\alpha_k = \exp T_k/\tau$, and T_k is the time of the electron motion between the collisions with the sample surface along the arc corresponding to the kth specular reflection channel.

The solution of Eq. (11)

$$F_i = W_{lk}A_k \frac{\operatorname{minor}\{\alpha_{li}\}}{\det\{\alpha_{jm}\}}, \qquad \alpha_{ik} = \alpha_i\delta_{ik} - W_{ik} \tag{12}$$

permits us to find the distribution of electron current at the sample surface. If the probability of choosing some specular reflection channels is greater than the probability for an electron to experience intrabulk scattering during the time T_k, i.e.,

$$W_{ik} \gg r/l \tag{13}$$

for any i and k, then W_{ik} is not included in the expression for F_i. In the case due to strong mixing of the states belonging to different channels of specular reflection, the asymptotic value of F_i becomes independent of i and equals

$$F_i = F = \gamma_0 \sum_{k=1}^{N} A_k, \qquad \gamma_0^{-1} \equiv \sum_{k=1}^{N} (\alpha_k - 1) \tag{14}$$

Simple calculations give the following result for the electric conductivity of the skin layer:

$$\sigma_{\text{skin}} = a\sigma_0 \frac{\langle(\Delta x)^2\rangle}{r^2\langle 1\rangle}, \qquad \Delta x = \sum_{k=1}^{N} \Delta x_k, \qquad a \simeq 1 \tag{15}$$

where Δx_k is the drift of a conduction electron during the time T_k along the electric current direction taken as the axis x. The axis z is directed along the magnetic field. The angle brackets denote averaging over FS.

The contribution to σ_{skin} of the electrons moving along complex nonperiodic trajectories after averaging over all the possible paths with the condition of Eq. (13) obeyed appears to be similar to that in the case of the strictly periodic motion when the electron inevitably passes through all the specular reflection channels with the drift $\Delta x = \sum_{k=1}^{N} \Delta x_k$ along the axis x per the period. They have opposite signs for the electron and hole states of conduction electrons, and the occurrence of the electron–hole Umklapp processes among the specular reflection channels decreases σ_{skin}. However, this fact does not influence the asymptotic behavior of the electric conductivity of the sample since in real metals there always are charge carriers whose reflection by the sample boundary is single channeled. But even with multichannel reflection Δx does not become zero at all the FS sections by the plane $p_z = \text{const}$. Therefore the electron–hole Umklapp processes are capable of reducing l_{eff}, not affecting, however, the dependence on a high magnetic field, and at $\tau \to \infty$ the effective mean free path of the charge carriers specularly reflected by the sample boundary also increases infinitely.

b. Metals with Open FS. The electric current in the sample is, as in compensated metals, conducted mainly by charge carriers belonging to closed FS sections and skipping along the sample surface. If during specular reflection an electron is able to jump from a closed FS section to an open one with the probability Q and leave the skin layer for the interior of the sample along an open trajectory, its effective mean free path in the bulky plate ($d \gg l$) tends to a finite value of r/Q at $\tau \to \infty$. In a thin plate the electron after drifting due to the Umklapp process from an open FS section to a closed one and then again to an open one may come back to the initial face of the plate. If during the time of this cycle equal in order of magnitude to $2d/v$, the probability of experiencing the interior scattering is less than Q, i.e., $Q \gg d/l$, then the effective mean free path of these electrons appears to be the same as in the case of strictly periodic motion (Fig. 2c). Since the maximum drift along the current direction during the period is about r, then during l/d cycles the electron drifts at the distance of about rl/d, i.e.,

$$l_{\text{eff}} = rl/d \tag{16}$$

This is also the effective mean free path of the electrons not leaving the open FS section which is formed by the electric conductivity of the core of the plate. The contribution of these electrons to the sample electric conductivity is similar to that made by a small portion (of about r/d) of the charge carriers forming the surface current. Equation (16) holds for l_{eff}, if the time during which the electron drifts from one face of the plate to the opposite one is not equal to the integer number of the periods T of motion along an open orbit. If

$$d = n \frac{cB}{eH} \cos \vartheta \tag{17}$$

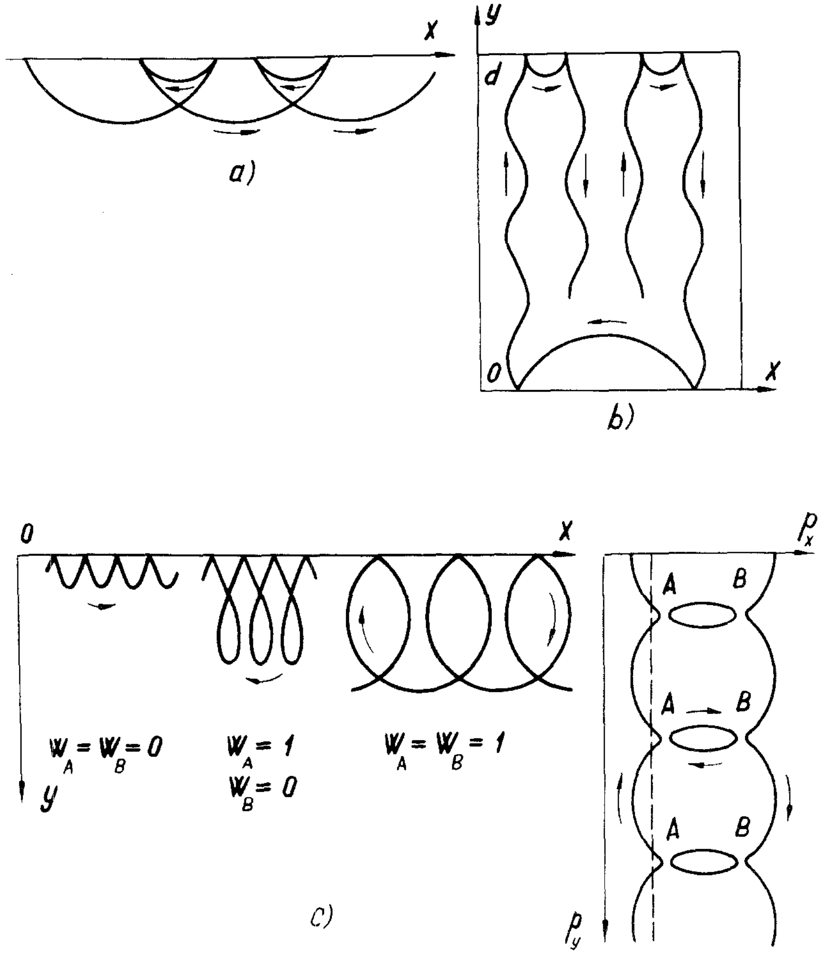

Fig. 2. Trajectories of conduction electrons skipping along the sample surface for multichannel specular reflection (a, b) and magnetic breakdown (c).

then after specular reflection from the opposite face the electron comes back to the initial face of the plate without any shift along the axis during the whole cycle. Thus, its effective mean free path is r rather than rl/d, and with Eq. (17) obeyed, the core current of the sample as a function of H changes periodically with the period

$$\Delta H = cB \cos \vartheta / ed \tag{18}$$

Here $B = p_\eta(t + T) - p_\eta(t)$ is the period of the open FS section, where the axis p_η is along the open orbit, and ϑ is the angle between the axes x and η.

c. Magnetic Breakdown. The charge carriers skipping along the plate face can leave the skin layer accomplishing a magnetic breakdown to the nearest orbit, which does not contact the sample boundaries. If after several revolutions along this orbit the electron returns to the trajectory broken by specular reflections, its drift along the direction of current, though weakened by the magnetic breakdown, may become infinitely large at $\tau \to \infty$. In sufficiently high magnetic fields, when the probability of magnetic breakdown of electrons in the region A and B (Fig. 2c) is appreciably nonzero, i.e., $W_A \gg r/l$ and $W_B \gg r/l$, their contribution to σ_{skin} appears to be similar to that for strictly periodic motion, when $W_A = W_B = 1$. In this case l_{eff} is already independent of magnetic field and the plate resistance is proportional to H, if $d < l^2/r$. For the configuration shown in Fig. 2c the calculation of l_{eff} with any W_A and W_B gives

$$l_{\mathrm{eff}} = l\frac{CW_A W_B + (r/l)(C_1 W_A + C_2 W_B) + (r/l)^2}{DW_A W_B + (r/l)(D_1 W_A + D_2 W_B) + (r/l)^2} \tag{19}$$

i.e., magnetic breakdown does not change the order of magnitude of the effective mean free path, $C, D, C_i, D_i \sim 1$.

If owing to magnetic breakdown the electron can drift through a chain of closed orbits from one face of the thin plate to the opposite one, then at $\tau \to \infty$ the effective mean free path is the same as for the electron drifting to the interior of the sample along an open trajectory. The analysis of this case is similar to taking into account the two-channel reflection of charge carriers from an imaginary conductor boundary parallel to the direction of the drift along magnetic breakdown trajectories.

Thus, multichannel processes of specular reflection do not affect the order of magnitude of the effective mean free path of conduction electrons. Only in bulky samples may the Umklapp process from closed FS sections to open ones appreciably reduce the effective mean free path of charge carriers.

3. The considerable decrease of the effective mean free path of conduction electrons interacting with the sample surface is connected with the surface roughness. The charge carrier scattering by not too rough surfaces may be taken into account using Falkovskii's boundary condition[17] at an averaged sample boundary at $y_s = 0, d$ generalized for an arbitrary number of specular reflection channels[18]

$$\psi_i(\mathbf{p}_+) - W_{ik}\psi_k(\mathbf{p}_-) = \int d^3p'\, w_{ik}(\mathbf{p}', \mathbf{p}_+)[\psi_k(\mathbf{p}') - \psi_k(\mathbf{p}_-)] \tag{20}$$

Integration over $\mathbf{p}'$ is made for all states of charge carriers incident onto the sample boundary and the indicatrix of their scattering is readily related to the correlation radius and rms height of roughness. The importance of the electron scattering by the surface to kinetic phenomena can be analyzed to a sufficient extent for two limiting cases, when the function ψ and the scattering indicatrix differ considerably in their sharpness. If the scattering indicatrix is more sharp, the Focker–Planck method is applicable and the boundary condition of Eq. (20) reduces to a second-order differential equation.[19] In the opposite limiting case the boundary condition has the form of Eq. (9), though normalization of the matrix components W_{ik} is different from unity

$$\sum_{i=1}^{N} W_{ik} = q(\tilde{\mathbf{p}}_k) \equiv q_k \tag{21}$$

The latter case, when the functionals of the indicatrix of charge carriers scattering $\chi_i(\mathbf{p}) = \int d^3p' \, w_{ik}(\mathbf{p}', \mathbf{p}) \, \psi_k(\mathbf{p}')$ are negligible and may be omitted in the boundary condition of Eq. (20), is similar to the approximation of the Fuchs specularity coefficient. It is easy to verify that the total probability of specular reflection q_k is the closer to unity, the smaller is the angle α between the sample surface and the velocity vector $\mathbf{v}(\tilde{\mathbf{p}}_k)$ of the incident electron, and at small α we have[19–22]

$$q(\alpha) = 1 - a\alpha, \qquad a = q'(0) \tag{22}$$

With a single-channel reflection of charge carriers by the sample boundary the solution of the kinetic equation has the form

$$\psi = \int_{\lambda_1}^{t} e\mathbf{v}(t') \, \mathbf{E}(\mathbf{r} + \mathbf{r}(t') - \mathbf{r}(t)) \exp\left(\frac{t'-t}{\tau}\right)$$
$$+ \frac{\int_{\lambda_1}^{\lambda_1 + T_1} e\mathbf{v}(t') \, \mathbf{E}(\mathbf{r}_1 - \mathbf{r}(\lambda_1) + \mathbf{r}(t')) \exp\left(\frac{t'-t-T_1}{\tau}\right) dt'}{1 - q\exp(-T_1/\tau)} \tag{23}$$

and simple calculations give the following expression for the effective mean free path of conduction electrons forming the skin layer:

$$l_{\text{eff}} = \frac{r}{W + r/l} \tag{24}$$

where W is the averaged value of $[1 - q(\alpha)]/\alpha$ obtained using the relation

$$\int_0^{\pi} \frac{\sin^3 \alpha \, d\alpha}{1 - q(\alpha) - \alpha(r/l)} \equiv \frac{1}{W + r/l} \int_0^{\pi} \frac{\sin^3 \alpha}{\alpha} \, d\alpha \tag{25}$$

It is readily shown that in the case of not too rough a surface with multichannel reflection the probability of the Umklapp processes $Q_k =$

$\sum_{i \neq k} W_{ik}$ breaking the periodicity of the motion of charge carriers is not included in the expression for F, if $Q_k \gg (\alpha_k - q_k)$. Then the function F has the form

$$F = \gamma q_i A_i, \qquad \gamma^{-1} \equiv \sum_{m=1}^{N} (\alpha_m - q_m) \tag{26}$$

and for l_{eff} in the absence of Umklapp processes sending the electron to an open FS section the following expression is valid:

$$l_{\text{eff}} = \frac{\langle (\Delta x)^2 \rangle}{\langle 1 \rangle \, r(W + r/l)} \tag{27}$$

and W has the same meaning as in Eq. (24).

If the electron incident on the sample boundary at the angle to its surface, jumps to the open FS section with the probability $Q(\alpha)$, then the effective mean free path of an electron in the bulky plate is

$$l_{\text{eff}} = r \int_0^{\pi} \frac{\sin^3 \alpha \, d\alpha}{1 - q(\alpha) + Q(\alpha) + \alpha(r/l)} \tag{28}$$

and in the thin plate the probability of skipping to the open FS section due to the Umklapp processes does not enter into the expression for l_{eff}, when $Q(\alpha) \gg [1 - q(\alpha) + d/l]$ most of angles α. The contribution of these electrons to the electric conductivity of the skin layer in the thin plate has the form

$$l_{\text{eff}} = \frac{r}{W + d/l} \tag{29}$$

and for sufficiently rough a sample surface, when $W \gg d/l$, the drifts to the open orbit due to the Umklapp processes are not of essential importance.

As the magnetic field makes a small angle θ with the plate surface, the maximum path of conduction electrons along the specular boundary is r/θ at $\tau = \infty$.[11] At $\tau \to \infty$ this path is the mean free path of the electrons skipping along the sample boundary. Hence, the above formulas for l_{eff} hold, if $1/l$ is replaced with $1/l + \theta/r$. In this case the specularity criterion of the metallic surface changes. If $W \ll \theta$, we do not distinguish such a surface from a specular one, and the surface state manifests itself only in a small correction for the conductor resistance.

In derivation of the above formulas for l_{eff} only the electric field E_x along the electric current direction is taken into account. This field may, with a sufficient accuracy, be considered homogeneous if the distance between conducting contacts exceeds significantly the mean free path of charge carriers. To make these formulas rigorous, it is necessary also to take into account the inhomogeneous electric field along the normal to the conductor surface, which should be found from the electroneutrality condition for metal.[11] A strict calculation, however, gives only a more exact numerical factor of about unity in the formulas for the effective mean free path of charge carriers interacting with the metallic surface.

4. Scattering of conduction electrons by roughness and impurity atoms at the sample surface has been studied in numerous theoretical works (e.g., Okulov and Ustinov's review[22] and the cited literature therein). The scattering indicatrix carries specific and detailed information on the state of the metallic surface. The only functional of the scattering indicatrix W, which can be found from the electroconductivity of the plate in the magnetin field parallel to its surface, does not give a complete idea of the surface profile. But together with the data on the oscillatory H dependence of the Hall field and magnetorisistance of thin conductors one succeeds in finding a set of various functionals of the indicatrix of charge carrier scattering by the metallic surface. In particular, in the magnetic field inclined to the plate surface the dependence of the probability of specular reflection upon the angle of the conduction electron incidence on to the sample boundary can be found for $\theta \gg r/d$.

The Sondheimer effect is quite sensitive to the shape of the conductor surface. If the plate surface consists of flat terraces parallel to each other and the magnetic field is orthogonal to the faces of the plate and the crystal symmetry plane, then, when the probability of an electron fall onto intermediate areas between terraces is low, the size effects are absent. This is connected with the fact that specular reflection by the flat area of the sample surface changes only the sign of the normal velocity component of the electron and its motion in the plate remains the same as in an unlimited conductor. The situation changes essentially if the magnetic field deviates from the normal to the plate surface or the orientations of the flat areas of the sample are different. In this case the Sondheimer oscillations occur even at locally specular reflection of charge carriers, and the oscillation amplitude is determined by the functionals of the scattering indicatrix, which are also dependent on the energy–momentum relationship for conduction electrons.

Similarly to the solution of the inverse problem of finding the electron energy spectrum, which was formulated by I. Lifshits and L. Onsager, it is possible to reconstruct the indicatrix of charge carrier scattering by the sample boundary from the galvanomagnetic characteristics of thin conductors at various orientations of the high magnetic field with respect to their surface. The set of experimental functionals of the scattering indicatrix of charge carriers is quite sufficient for solution of this problem, if the energy–momentum relationship for conduction electrons is known.

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